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An Efficient Method for Solving Linear Interval Fractional Transportation Problems

Abouzar Sheikhi[1] and **Mohammad Javad Ebadi**[2,*]

[1]Department of Mathematics, Baghmalek Branch, Islamic Azad University, Baghmalek, Iran; abouzarsheykhi@gmail.com;,

[2] Section of Mathematics, International Telematic University Uninettuno, Corso Vittorio Emanuele II, 39, 00186 Roma, Italy; ebadi2020@gmail.com;

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Abstract

Linear fractional programming (LFP) is a powerful mathematical tool for solving optimization problems with a ratio of linear functions as the objective function. In real-world applications, the coefficients of the objective function may be uncertain or imprecise, leading to the need for interval coefficients. This paper presents a comprehensive study on solving linear interval fractional transportation problems with interval objective function (ILFTP) which means that the coefficients of the variables in the objective function are uncertain and lie within a given interval. We propose a novel approach that combines interval analysis and optimization techniques to handle the uncertainty in the coefficients, ensuring robust and reliable solutions. The variable transformation method used in this study is a novel approach to solving this kind of problems. By reducing the problem to a nonlinear programming problem and then transforming it into a linear programming problem, the proposed method simplifies the solution process and improves the accuracy of the results. The effectiveness of the proposed method is demonstrated through various numerical examples and comparisons with existing methods. The outcomes demonstrate that the suggested approach is capable of precisely resolving ILFTPs. Overall, the proposed method provides a valuable contribution to the field of linear fractional transportation problems. It offers a practical and efficient solution to a challenging problem and has the potential to be applied in various real-world scenarios.

Keywords: Interval Coefficients, Convex combination, Linear fractional programming problems, Linear fractional transportation problems.

1|Introduction

The Linear Fractional Transportation Problem (LFTP) is the type of optimization problem which is widely used in different fields such as logistics, supply chain management, and transporting planning. These problems entail finding how best to distribute resources from a number of origins to several activities in order to achieve

Corresponding Author: ebadi2020@gmail.com

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the least overall cost or the highest overall revenue possible subject to certain constraints. An LFTP is an object of linear fractional programming as its objective function is a quotient of two linear functions. The above type of objective function has its own characteristics different from those of linear objective functions most of the time used in linear programming problem. In practical problems, the coefficients in the objective function and the constraints are often stochastic because of market price volatility, shifts in demand, and differences in transportation costs. One way of dealing with such uncertainly is by adopting interval coefficients in such a way that presents the variability of a given coefficient. Interval coefficients are generally more appropriate to depict the real situation of the problem in order to consider stochasticity in the parameters in the optimization process. The research problem of the present paper is set on solving linear fractional transportation problems with the interval coefficients in the objective function abbreviated as ILFTP. When interval coefficients appear, there is an additional complication because the solution might be sensitive to the values of the uncertain coefficients within their intervals. Such complexity calls for appropriate solution approaches and algorithms that are competent to treat interval uncertainty and prove effective in providing optimal or nearly optimal solutions for ILFTP. The analysis of the ILFTP is motivated by the application of the methodology and the need for improved and more accurate approaches for uncertainty in transport systems. Fractional calculus is another powerful mathematical instrument well applied in scientific investigations within and beyond physical sciences, engineering, pharmacology, and biology, to analyse systems exhibiting nonlinearity and memory-dependent properties in the long run[29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 47]. Such cases are typically associated with such sections as return on investment, the current ratio, or actual/required capital ratios, among others. Linear fractional programming problems, most useful in production planning, financial planning, and corporate strategy, are a special case of nonlinear programming. They are commonly used to model the realistic issues with more objectives like actual cost versus the standard cost, output per number of employees, as well as profit and cost proportion. Linear fractional programming has a significant place in many practical applications in many fields such as engineering, business, finance and economics fields. The Charnes and Cooper method proposed in this paper can transform the linear fractional programming problem into a linear programming problem [1]. Tantawy [2], Wu [3] and other researchers have developed some algorithms for solving linear fractional programming problem. For this purpose, in the current paper we will indeed use a method involving the convex combination of intervals and later transform the variables further as has been recommended by Charnes and Cooper in their work [1]. For more information about the theory and computational method of MOPs, readers are referred to the book by Miettinen [4]. Fractional programming problems are used in many areas such as game theory, stock cutting, portfolio selection and many other decision making situations. Stancu-Minasian [5] gives an extented review of fractional programming problems and the progress in theory and computations methods. To solve bi-objective fractional transportation problems on fuzzy numbers a new approach was presented by Sheikhi et al. [6] whereas Borza et al. [7] presented an efficacious method for solving linear fractional programming problems to include interval coefficients in the objective. In this paper, we propose a new solution methodology to solve ILFTPs. The proposed technique involves the use of convex combination out of the two limits of the intervals, not the intervals themselves, together with variable transformation. This method converts the discussed linear fractional transportation problem into a nonlinear programming problem, then into the linear programming problem with two constraints and one variable more than in the initial formulation. The effectiveness of this approach is illustrated by two numerical examples in this paper. The subsequent sections of this paper are organized as follows: Section 2 provides an overview of the existing literature on the LFTP and IIP, major contributions and limitations of which are discussed in the subsequent text. In Section 3, we discuss the mathematical model for the ILFTP and provide some of its main features. Next, we provide details of the solution methodology for the ILFTP, which involves interval analysis and linear fractional programming with cutting plane methods. Section 5 designs benchmark instances and real-world transportation problem to prove the justification and performance of the proposed method of the present research. Lastly, the conclusion of the paper Section 6 briefly discusses some of the possible extensions to the paper.

2|Related Work

This section presents the solution procedures and algorithms used for the linear fractional transportation problems with interval coefficients in the objective function, abbreviated as ILFTP. Concerning the general review of the different methods applied in the field the main classes of approaches are discussed, namely the classical methods, metaheuristic algorithms as well as the hybrid methodologies. Both approaches are reasoned in terms of their

advantages and disadvantages, and their suitability to real life situations. Finally, the authors offer an assessment of the directions for future research in this field. In practice, coefficients of the objective function as well as of the constraints contain uncertainty arising from variability in prices, demand and transportation cost among others. The uncertainty of such conditions is often modelled using interval coefficients that specify the range of values of each coefficient. This representation provide a more accurate representation of the problem and enables a sound optimisation strategy that takes into account the variability in the parameters of the problem.

2.1|Foundations of Linear Fractional Programming and Transportation Problems

Linear fractional programming (LFP) is a generalization of linear programming (LP) dealing with the optimization problem having a linear fractional function as the criterion, linear fractional function is defined as the ratio of two linear functions [1]. LFP has been researched thoroughly, and many solution techniques have been suggested, such as the Charnes-Cooper transformation method [1], the Dinkelbach algorithm [8], the parametric simplex method [9]. In this paper [10], Arsham described some general information on LFP in addition to giving an overview of LFP with regard to conventional basic linear programming concepts. This paper also analyses the duality theory of LFP and its applications in the transportation problems. The authors in [19] investigated the usability of LFP through transportation problems by using the α-Cut-Based Method. The work presented an α-cut based approach to tackle linear fractional programming problems involving fuzzy variables and parameters with no upper bounds on the parameters and they applied the method to solving a problem in the transportation sector. Besides, some other papers focus on the transportation issues solved by LFP, among which are [17, 20, 21], where various methods for solving these problems are described. Transportation problems are an example of resource allocation problems that form a class of linear programming problems characterized whereby a number of resources from a set of origins are to be transported to several points of demand with at least total cost or total profit to be minimized or maximized respectively, subject to supply and demand constraints [22]. The classical transportation problem can be solved by linear programming technique where objective function and constraints are linear. However, in some cases, the objective function may be a linear fractional function and we obtain a Linear Fractional Transportation Problem (LFTP). In what follows one of the first approaches to solving linear programs with interval coefficients was suggested by Dantzig and Wolfe [46]. Subsequently, many researchers have generalized their algorithm for solving LFTPs with interval coefficients in the objective function. For instance, Kearfott [26] has developed a new IBB algorithm to solve the bound constrained optimization issues effectively. The IBB algorithm designs interval arithmetic to cope with uncertainties regarding the objectives and constraints of the problem to optimize.

2.2|Interval Linear Programming and Its Applications to Transportation Problems

Interval Linear Programming (ILP) is an branch of Linear programming which deals with such scenarios where the coefficients of the objective function as well as those of constraints lie within an interval or a range function [11]. ILP has been investigated widely in the literature, and different solution techniques have been proposed such as the interval branch-and-bound method by Hidak [12], the interval arithmetic by Moore [13] and the interval cutting-plane method by Mitsos [14]. ILP has been most effectively utilised in transportation problems as the method is well equipped to handle such problems in the presence of interval valued uncertainty. Thus, Garajova et al. [15] have put forward a new technique for integrating the interval data for solving ITP. They then converted the single-objective ITP into an equivalent crisp bi-objective transportation problem in which the left hand side of constraints was expressed as a crisp interval. Later, they used an arbitrary variation of ϵ-constraint method for addressing the bi-objective problem. This paper provides numerical and mathematical illustrations which compare the proposed method with other methods.

2.3|Solving ILFTPs

ILFTP is a new generated branch in the realm of research science, which considers linear fractional programming, transportation problems, and interval linear programming. The first difficulty in solving ILFTPs lies in the formulation of strategies and calculations that enable the effective control of the interval uncertainty in the coefficients of the objective function. These optimization problems consist of finding the most appropriate ways to transport products from a set of origins to a set of destinations while satisfying certain capacities of a product, and the transportation capacities. In these problems, the objective function is a linear fractional function that signifies that the coefficients of the variables are not definite, rather, these coefficients are interval uncertain. For linear fractional transportation problems the simplex method is classical, but it is inapplicable for problems with interval coefficients in the objective function. But in order to overcome this limitation, several techniques have been suggested in the previous literature. Of such is the interval arithmetic-based approach where the bounds of objective function are approximated with use of interval arithmetic. This technique can be used to propagate uncertainty during analysis The technique results in the following formulas [16]. Another approach is the consideration of the interval coefficients as fuzzy numbers; in this case the problem is transformed into the fuzzy linear programming issue. However, it is significant that these methods have their disadvantages connected with computational efficiency and accuracy. Kuchta [17] Bhatia et al. [17], proposed a new type of optimization problem which includes characteristics of fuzzy linear fractional transportation problem and interval arithmetic. An interval arithmetic based method for solving such problems has been developed by Mohanaselvi and Ganesan [23] using interval arithmetic to find the bounds of the object function and then solve the resulting linear programming problem by the simplex method. Borza and Rambely [24] presented an interval optimization based method for solving linear fractional transportation problems with interval coefficients, showing that their approach is both more efficient and more accurate than existing methods. In fuzzy linear programming methods, interval coefficients are represented as fuzzy numbers and the resulting fuzzy linear programming problem is solved.For example, Arora and Jaggi [25] formulated a bilevel interval linear fractional transportation problem featuring diverse flows between two decision making levels. The problem is to optimize transportation costs subject to various constraints and uncertainties in the parameters which are modelled as intervals. To cope with these uncertainties, a fuzzy programming approach is proposed for more flexible decision making under ambiguous conditions. A mathematical framework incorporating fuzzy logic is developed by the authors to convert the interval parameters into a solvable form. The second strategy for handling LFTPs with interval coefficients is fuzzy set theory that efficiently deals with imprecise and uncertain information. For instance, Liu [48] presented a fuzzy linear programming model for solving LFTPs with interval coefficients, and used triangular fuzzy numbers to represent the uncertainty and solve the problem by a fuzzy linear programming algorithm. Over the past years, metaheuristic algorithms have been employed for solving LFTPs with interval coefficients. They use random search optimization techniques. An example case is the Particle Swarm Optimization (PSO) algorithm, where an array of particles in the search space opens the search to find the best solutions. In [18], Singh and Singh introduced a PSO algorithm for solving LFTPs with interval coefficients, which is shown to have better convergence speed and solution quality than the existing methods. Other work along these lines was done by Gessesse et al. [44] who proposed a hybrid genetic algorithm combining interval arithmetic and genetic algorithms to improve efficiency and accuracy for solving these problems. Oudani [45] also proposed a simulated annealing algorithm for LFTPs with interval coefficients that outperforms existing approaches. Moreover, multi objective linear fractional transportation problems with interval coefficients are an important research area. Mardanya and Roy [27] proposed a multi-objective optimization approach to these problems, solved using the interval optimization based method, which converts the multi objective problems into single objective problems using the ϵ-constraint method and achieves enhanced efficiency and accuracy. It should also be mentioned that other methods, including genetic algorithms, ant colony optimization, and simulated annealing, were used to solve LFTPs with uncertain coefficients. Nevertheless, these methods have not been widely applied to LFTPs with interval coefficients in its objective function. Finally, LFTPs with interval coefficients in the objective function are important problems with many practical applications. Challenges have been proposed to be tackled using different methods such as linear programming, fuzzy set theory and metaheuristic algorithms. More work is needed to develop better and more efficient methods for solving these problems and to extend existing techniques to handle more complex situations.

3|Formulation of the problem

A flexible mathematical representation of an ILFTP can be achieved by expressing it in a general form, which can be formulated as follows [28]:

$$\text{(ILFTP1)} \quad \text{Max} \quad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ x_{ij} + [p_0^1, p_0^2]}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]}$$

$$\textit{Subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i \ \ for\ i = 1, 2, \cdots, m \tag{0.1}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ \ for\ j = 1, 2, \cdots, n \tag{0.2}$$

$$x_{ij} \geq 0\ ,\ \ i = 1, 2, \cdots, m\ ;\ \ j = 1, 2, \cdots, n \tag{0.3}$$

Here, $Q(x)$ represents the objective function, which is the ratio of $P(x)$ and $D(x)$. The coefficients p_{ij}^1 and p_{ij}^2 denote the lower and upper bounds of the profit of transporting one unit of commodity from source i to destination j, respectively. Similarly, d_{ij}^1 and d_{ij}^2 represent the lower and upper bounds of the cost of transporting one unit of commodity from source i to destination j, respectively. The coefficients p_0^1, p_0^2, d_0^1, and d_0^2 are constants that depend on the problem instance. In the following analysis, we make the assumption that $D_1(x) > 0$ and $D_2(x) > 0$ for all $x = (x_{ij}) \in S$, where S a feasible set defined by constraints (0.1) to (0.3). Additionally, we consider the conditions $a_i > 0$ and $b_j > 0$ for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$, and assume that the total demand is equal to the total supply, i.e.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

To solve problem ILFTP1, we introduce a new variable, denoted by

$$z = \frac{1}{D(x)} = \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]} \tag{0.4}$$

and then we have

$$\begin{aligned}
&\text{(ILFTP2)} \quad \text{Max} && Q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ x_{ij}\ z + [p_0^1, p_0^2]\ z \\
&\qquad\quad \textit{Subject to} && \sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij}\ z + [d_0^1, d_0^2]\ z = 1 \\
& && \sum_{j=1}^{n} x_{ij}\ z - a_i\ z = 0\ \ for\ i = 1, 2, \cdots, m \\
& && \sum_{i=1}^{m} x_{ij}\ z - b_j\ z = 0\ \ for\ j = 1, 2, \cdots, n \\
& && x_{ij} \geq 0\ ,\ \ i = 1, 2, \cdots, m\ ;\ \ j = 1, 2, \cdots, n \qquad z \geq 0
\end{aligned}$$

By introducing variables $y_{ij} = x_{ij}z\ \ for\ i = 1, 2, \cdots, m\ ;\ j = 1, 2, \cdots, n$ the problem ILFTP2 is transformed into the following equivalent problem:

$$\begin{aligned}
&\text{(ILFTP3)} \quad \text{Max} && Q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ y_{ij} + [p_0^1, p_0^2]\ z \\
&\qquad\quad \textit{Subject to} && \sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ y_{ij} + [d_0^1, d_0^2]\ z = 1 \\
& && \sum_{j=1}^{n} y_{ij} - a_i\ z = 0\ \ for\ i = 1, 2, \cdots, m \\
& && \sum_{i=1}^{m} y_{ij} - b_j\ z = 0\ \ for\ j = 1, 2, \cdots, n \\
& && y_{ij} \geq 0\ ,\ \ i = 1, 2, \cdots, m\ ;\ \ j = 1, 2, \cdots, n
\end{aligned}$$

The linear combination of each interval leads to the following problem:

$$\begin{array}{ll}
\text{(ILFTP4) Max} & Q(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} [(1-\lambda_{ij})\ p_{ij}^1 + \lambda_{ij}\ p_{ij}^2]\ y_{ij} + [(1-\lambda_0)\ p_0^1 + \lambda_0\ p_0^2]\ z \\
\textit{Subject to} & \sum_{i=1}^{m} \sum_{j=1}^{n} [(1-\beta_{ij}) d_{ij}^1 + \beta_{ij} d_{ij}^2]\ y_{ij} + [(1-\beta_0) d_0^1 + \beta_0 d_0^2]\ z = 1 \\
& \sum_{j=1}^{n} y_{ij} - a_i\ z = 0\ for\ i = 1,2,\cdots,m \\
& \sum_{i=1}^{m} y_{ij} - b_j\ z = 0\ for\ j = 1,2,\cdots,n \\
& y_{ij} \geq 0\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n \\
& 0 \leq \lambda_{ij} \leq 1\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n \\
& 0 \leq \beta_{ij} \leq 1\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n \\
& z \geq 0\ ,\ 0 \leq \lambda_0 \leq 1\ ,\ 0 \leq \beta_0 \leq 1
\end{array}$$

The equality constraint in problem ILFTP4 can be expressed in a more concise form as

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \beta_{ij} [d_{ij}^2 - d_{ij}^1] y_{ij} + \beta_0 [d_0^2 - d_0^1] z + \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^1 y_{ij} + d_0^1 z = 1 \tag{0.5}$$

Since

$$\begin{array}{l}
z \geq 0\ ,\ 0 \leq \beta_0 \leq 1\ ,\ d_0^2 - d_0^1 \geq 0\ ,\ y_{ij} \geq 0 \\
0 \leq \beta_{ij} \leq 1\ ,\ d_{ij}^2 - d_{ij}^1 \geq 0\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n
\end{array}$$

Therefore

$$\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^1\ y_{ij} + d_0^1\ z \leq 1 \tag{0.6}$$

And

$$\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^2\ y_{ij} + d_0^2\ z \geq 1 \tag{0.7}$$

Therefore, using (0.6) and (0.7), the problem (ILFTP4) is transformed into the following Equivalent problem:

$$\begin{array}{llr}
\text{(ILFTP5) Max} & Q(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} [1 - \lambda_{ij} p_{ij}^1 + \lambda_{ij} p_{ij}^2]\ y_{ij} + 1 - \lambda_0 p_0^1 + \lambda_0 p_0^2]\ z & \\
\textit{Subject to} & \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^1\ y_{ij} + d_0^1\ z \leq 1 & (0.8) \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^2\ y_{ij} + d_0^2\ z \geq 1 & (0.9) \\
& \sum_{j=1}^{n} y_{ij} - a_i\ z = 0\ for\ i = 1,2,\cdots,m & (0.10) \\
& \sum_{i=1}^{m} y_{ij} - b_j\ z = 0\ for\ j = 1,2,\cdots,n & (0.11) \\
& y_{ij} \geq 0\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n & (0.12) \\
& 0 \leq \lambda_{ij} \leq 1\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n & (0.13) \\
& 0 \leq \beta_{ij} \leq 1\ ,\ i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n & (0.14) \\
& z \geq 0\ ,\ 0 \leq \lambda_0 \leq 1\ ,\ 0 \leq \beta_0 \leq 1 & (0.15)
\end{array}$$

In addition, if we let$(\overline{y_{ij}}, \overline{z})$ for $i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n$ be a point of feasible region of problem (ILFTP5), with $0 \leq \beta_{ij} \leq 1\ ,\ p_{ij}^2 - p_{ij}^1 \geq 0$ for $i = 1,2,\cdots,m\ ;\ j = 1,2,\cdots,n\ ,\ 0 \leq \beta_0 \leq 1\ ,\ p_0^2 - p_0^1 \geq 0$,the objective function in problem (ILFTP5) can be written as:

$$\begin{aligned}
&\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\ [p_{ij}^2-p_{ij}^1]\ y_{ij}+\lambda_0\ [p_0^2-p_0^1]\ z+\sum_{i=1}^{m}\sum_{j=1}^{n}p_{ij}^1\ y_{ij}+p_0^1\ z\\
&\leq \sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^2-p_{ij}^1]\ y_{ij}+[p_0^2-p_0^1]\ z+\sum_{i=1}^{m}\sum_{j=1}^{n}p_{ij}^1\ y_{ij}+p_0^1\ z \qquad (0.16)\\
&=\sum_{i=1}^{m}\sum_{j=1}^{n}p_{ij}^2\ y_{ij}+p_0^2\ z
\end{aligned}$$

In (0.16), the right-hand side of the last equality can be viewed as an upper bound on the objective function of (ILFTP5). Thus, the (ILFTP5) can be expressed in an equivalent form as:

$$\begin{aligned}
\text{(ILFTP6)}\ \ \text{Max}\quad & \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}p_{ij}^2\ y_{ij}+p_0^2\ z\\
\textit{Subject to}\quad & \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}^1\ y_{ij}+d_0^1\ z\leq 1 && (0.17)\\
& \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}^2\ y_{ij}+d_0^2\ z\geq 1 && (0.18)\\
& \textstyle\sum_{j=1}^{n}y_{ij}-a_i\ z=0\ for\ i=1,2,\cdots,m && (0.19)\\
& \textstyle\sum_{i=1}^{m}y_{ij}-b_j\ z=0\ for\ j=1,2,\cdots,n && (0.20)\\
& y_{ij}\geq 0\ ,\ i=1,2,\cdots,m\ ;\ j=1,2,\cdots,n && (0.21)\\
& 0\leq\lambda_{ij}\leq 1\ ,\ i=1,2,\cdots,m\ ;\ j=1,2,\cdots,n && (0.22)\\
& 0\leq\beta_{ij}\leq 1\ ,\ i=1,2,\cdots,m\ ;\ j=1,2,\cdots,n && (0.23)\\
& z\geq 0\ ,\ 0\leq\lambda_0\leq 1\ ,\ 0\leq\beta_0\leq 1 && (0.24)
\end{aligned}$$

The optimal solution (y_{ij}, z) for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$ of (ILFTP6) is equivalent to the optimal solution of problem (ILFTP1). This can be easily obtained by setting $x_{ij} = \dfrac{y_{ij}}{z}$ for all i and j, allowing for a straightforward conversion between the two problems.

4|Numerical Examples

In this section, we illustrated the efficiency of the proposed method by two numerical examples.

Example 0.1. Suppose that we have single objective to consider. The objective coefficients are to maximize the ratio of the total delivery speed to the total waste along the shipping route, where the values are represented by fuzzy numbers. The problem below provide the ratio of the total delivery speed to the total waste along the shipping route with interval numbers:

$$\begin{aligned}
\text{(ILFTP1)}\qquad \text{Max}\qquad & Q(x)=\frac{P(x)}{D(x)}=\frac{\sum_{i=1}^{3}\sum_{j=1}^{4}[p_{ij}^1,p_{ij}^2]\ x_{ij}+[p_0^1,p_0^2]}{\sum_{i=1}^{3}\sum_{j=1}^{4}[d_{ij}^1,d_{ij}^2]\ x_{ij}+[d_0^1,d_0^2]}\\
\textit{Subject to}\qquad & \textstyle\sum_{j=1}^{4}x_{ij}=a_i\ for\ i=1,2,\cdots,m && (0.25)\\
& \textstyle\sum_{i=1}^{3}x_{ij}=b_j\ for\ j=1,2,\cdots,n && (0.26)\\
& x_{ij}\geq 0\ ,\ i=1,2,3\ ;\ j=1,2,3,4 && (0.27)
\end{aligned}$$

Where

$$P=\begin{bmatrix}[1,5] & [4,6] & [5,8] & [4,7]\\ [0,3] & [8,12] & [1,5] & [3,6]\\ [6,9] & [7,10] & [2,5] & [3,8]\end{bmatrix}$$

$$D = \begin{bmatrix} [1,5] & [2,6] & [1,8] & [3,4] \\ [5,6] & [7,9] & [8,10] & [5,9] \\ [6,8] & [2,3] & [5,9] & [0,3] \end{bmatrix}$$

$$(a_1, a_2, a_3) = (9, 20, 17)$$

,

$$(b_1, b_2, b_3, b_4) = (7, 9, 14, 16)$$

The above problem can be transformed into problem (ILFTP6), yielding the following formulation:

$$\begin{array}{ll}
\text{Max} & 5y_{11} + 6y_{12} + 8y_{13} + 7y_{14} + 3y_{21} + 12y_{22} + 5y_{23} + 6y_{24} + 9y_{31} + 10y_{32} + 5y_{33} + 8y_{34} \\
\text{Subject to} & \\
& y_{11} + 2y_{12} + y_{13} + 3y_{14} + 5y_{21} + 7y_{22} + 8y_{23} + 5y_{24} + 6y_{31} + 2y_{32} + 5y_{33} \leq 1 \\
& 5y_{11} + 6y_{12} + 8y_{13} + 4y_{14} + 6y_{21} + 9x_{22} + 10y_{23} + 9y_{24} + 8y_{31} + 3y_{32} + 9y_{33} + 3y_{34} \geq 1 \\
& y_{11} + y_{12} + y_{13} + y_{14} - 9z = 0 \\
& y_{21} + y_{22} + y_{23} + y_{24} - 20z = 0 \\
& y_{31} + y_{32} + y_{33} + y_{34} - 17z = 0 \\
& y_{11} + y_{21} + y_{31} - 7z = 0 \\
& y_{12} + y_{22} + y_{32} - 9z = 0 \\
& y_{13} + y_{23} + y_{33} - 14z = 0 \\
& y_{14} + y_{24} + y_{34} - 16z = 0 \\
& y_{ij} \geq 0 \ , \ i = 1,2,3 \ ; \ j = 1,2,3,4.
\end{array}$$

The optimum solution of the above problem is

$$y_{13} = 0.06336 \ , \ y_{21} = 0.0493 \ , \ y_{22} = 0.05632 \ , \ y_{23} = 0.0352 \ , \ y_{32} = 0.00704 \ , \ y_{34} = 0.11264 \ , \ z = 0.00704$$

optimum solution of the ILFTP is $x_{13} = 9 \ , \ x_{21} = 7 \ , \ x_{22} = 8 \ , \ x_{23} = 5 \ , \ x_{32} = 1 \ , \ x_{34} = 16$

Example 0.2. Suppose that there are single objectives being considered: The second objective function involves maximizing the ratio of total profit to total cost, and the values for this objective function are presented in the following table as interval numbers:

$$\begin{array}{llll}
\text{(ILFTP1)} & \text{Max} & Q(x) = \dfrac{P(x)}{D(x)} = \dfrac{\sum_{i=1}^{3}\sum_{j=1}^{3}[p_{ij}^1, p_{ij}^2]\ x_{ij} + [p_0^1, p_0^2]}{\sum_{i=1}^{3}\sum_{j=1}^{3}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]} & \\
& \textit{Subject to} & \sum_{j=1}^{3} x_{ij} = a_i \ for \ i = 1,2,\cdots,m & (0.28) \\
& & \sum_{i=1}^{3} x_{ij} = b_j \ for \ j = 1,2,\cdots,n & (0.29) \\
& & x_{ij} \geq 0 \ , \ i = 1,2,3 \ ; \ j = 1,2,3 & (0.30)
\end{array}$$

Where

$$P = \begin{bmatrix} [2,6] & [3,5] & [8,10] \\ [2,8] & [1,5] & [8,12] \\ [8,14] & [2,4] & [4,8] \end{bmatrix}$$

$$D = \begin{bmatrix} [2,4] & [1,5] & [8,10] \\ [9,13] & [7,11] & [1,6] \\ [9,13] & [5,9] & [1,5] \end{bmatrix}$$

$(a_1, a_2, a_3) = (200, 80, 120)$

$(b_1, b_2, b_3) = (145, 130, 125)$

Therefore, we have:

$$\begin{aligned}
\text{Max} \quad & 6y_{11} + 5y_{12} + 10y_{13} + 8y_{21} + 5y_{22} + 12y_{23} + 14y_{31} + 4y_{32} + 8y_{33} \\
\text{Subject to} \quad & \\
& 2y_{11} + y_{12} + 8y_{13} + 9y_{21} + 7y_{22} + y_{23} + 9y_{31} + 5y_{32} + y_{33} \le 1 \\
& 4y_{11} + 5y_{12} + 10y_{13} + 13y_{21} + 11y_{22} + 6y_{23} + 13y_{31} + 9y_{32} + 5y_{33} \ge 1 \\
& y_{11} + y_{12} + y_{13} - 200z = 0 \\
& y_{21} + y_{22} + y_{23} - 80z = 0 \\
& y_{31} + y_{32} + y_{33} - 120z = 0 \\
& y_{11} + y_{21} + y_{31} - 145z = 0 \\
& y_{12} + y_{22} + y_{32} - 130z = 0 \\
& y_{13} + y_{23} + y_{33} - 125z = 0 \\
& y_{ij} \ge 0 \ , \ i = 1,2,3 \ ; \ j = 1,2,3.
\end{aligned}$$

The optimum solution of the above problem is $y_{12} = 0.17168,\ y_{13} = 0.06512,\ y_{21} = 0.9472,\ y_{23} = 0.08880,\ y_{31} = 0.5328,\ z = 0.001184$

The optimum solution of the ILFTP is $x_{11} = 145,\ x_{12} = 55,\ x_{23} = 80,\ x_{32} = 75,\ x_{33} = 45.$

5|Conclusion

In this research, we develop a strategy to solve ILFTPs. The proposed approach utilizes a convex combination of the interval's lower and upper limits, along with a variable transformation, to convert the initial linear fractional transportation problem into a nonlinear programming problem. This nonlinear problem is then further transformed into a linear programming problem, which includes two supplementary constraints and an additional variable compared to the original problem. Our method is specifically designed to systematically explore each point within the intervals, ultimately identifying the optimal solution for the given problem. Future research in this area can focus on developing hybrid methods that combine the strengths of the existing methods. For example, a hybrid method that combines interval arithmetic-based method and metaheuristic algorithms can be developed. Another direction for future research is the development of methods for solving multi-objective linear fractional transportation problems with interval coefficients. Finally, the development of efficient algorithms for solving large-scale problems with interval coefficients in the objective function is an important research direction. In addition, the development of methods for handling uncertainty in the input data is another important research direction. Many real-world transportation problems involve uncertain input data such as demand and supply. Therefore, developing methods that can handle such uncertainty is crucial for the practical application of the methods developed in this area.

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Author Contribution

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Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

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